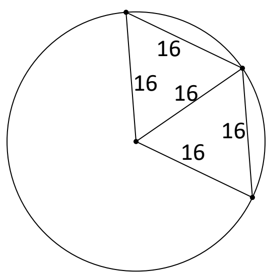
Answers

1. D
2. B
3. B
4. E (8)
5. C
6. A
7. A
8. D
9. D
10. B
11. A
12. A
13. A
14. C
15. A
16. B
17. D
18. C
19. D
20. B
21. B
22. C
23. A
24. A
25. D
26. B
27. E (58/3)
28. B
29. C
30. C

Solutions

1. From the given equation, we can tell that this will generate an ellipse with major axis on the x-axis. We can find the vertices are $(8/7, 0°)$ and $(8, 180°)$ and the x-coordinate of the center is $-24/7$. The distance between the centers is 64/7, so the distance from center to vertex is 32/7. In this form, we are guaranteed one focus at the pole, so the distance from center to focus is 24/7. The area of the ellipse is $(32/7)(8\sqrt{7}/7)\pi \to 256\sqrt{7}/49$.

2.



$$A = 2\left(\frac{16^2\sqrt{3}}{4}\right)$$
$$A = 128\sqrt{3}$$

3. Since the limit would evaluate to 0/0 L'Hopital's rule can be applied. Taking the derivative of the numerator and using the 2nd Fundamental Theorem of Calculus gives $\frac{1}{2}x^2$ and just 1 for the denominator. Evaluating this as x approaches 25 gives a final answer of 625/2.

4. The number of handshakes can be found by $H = (n)(n-1)/2$, where n is the number of boys (or girls). For n=2, H=1; for n=3, H = 3; for n=4, H=6; for n=5, H=10; for n=6, H=15; for n=7, H=21; for n=8, H=28. After n=8, the numbers are too large. The only pairs that sum to 31 are 21+10 and 28+3. Since we need the maximum number of boys, there must have been 8 boys to produce 28 handshakes.

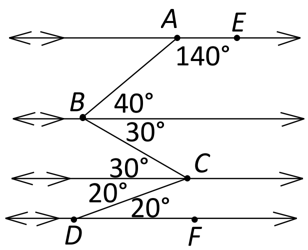
5. Using quotient rule results in $'(x) = \frac{(3-x)10(2x-4)^4-(2x-4)^5(-6(3-x)^5)}{(3-x)^{12}} = \frac{(6+2x)(2x-4)^4}{(3-x)^7}$. Plugging in 1 for x gives a result of 1.

6. $A = \frac{1}{2}\sqrt{2a^2+2b^2-c^2}$ and $B = \frac{2}{a+b}\sqrt{abs(s-c)}$, where s = semiperimeter

$$A = \frac{1}{2}\sqrt{2(4)^2+2(6)^2-8^2} = \sqrt{10} \text{ and } B = \frac{2}{4+8}\sqrt{4\cdot 8\cdot 9(9-6)} = 2\sqrt{6} \to \frac{B}{A} = \frac{2\sqrt{15}}{5}$$

7. Let d be the common difference between the roots, and let $a-d$, a, and $a+d$ be the three roots. The sum of these roots gives $3a$=900/125, using Vieta's formula, so a=12/5. Again using a Vieta formula, the product $(12/5 - d)(12/5)(12/5 + d)$=756/125, giving $d = \pm 9/5$. This gives roots of 3/5, 12/5, and 21/5. The sum of the two smallest roots is 15/5 = 3.

8.



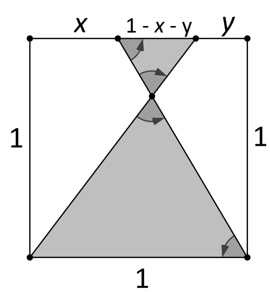
$x = 70°$

9. When the tank's volume is 36π its height is 4. $V' = 9\pi h'$ so evaluating the volume rate and solving for h' gives an answer of $\frac{1}{5}$ or 0.2.

10. $\frac{x+4}{x-2} - \frac{x+1}{x+2} < 1 \to \frac{(x^2+6x+8)-(x^2-x-2)-(x^2-4)}{(x-2)(x+2)} < 0 \to \frac{x^2-7x-14}{(x-2)(x+2)} > 0.$ $x = \frac{7 \pm \sqrt{105}}{2}$.
$b=-2,\ c=\frac{7-\sqrt{105}}{2},\ d=2,\ e=\frac{7+\sqrt{105}}{2}.$ $c+e-d^{-b} \to 7-2^{-2} \to 7-\frac{1}{4}=\frac{27}{4}$.

11. The distance between the parabola and the point can be expressed as $D^2 = (y^2)^2 + (y+3)^2$. Taking the derivative gives $2DD' = 4y^3 + 2y + 6$. Setting this equal to zero and solving gives a y value of -1. The x-coordinate would then be 1.

12. A

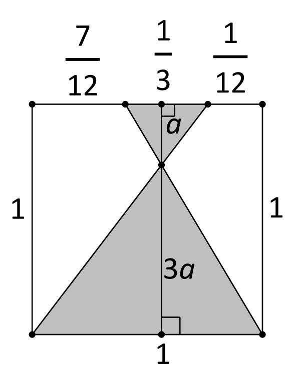


The shaded triangles are similar.

$\frac{\text{Area Large Shaded Triangle}}{\text{Area Small Shaded Triangle}} = \frac{9}{1}$

Ratio of the bases $\frac{3}{1}$

$\frac{3}{1} = \frac{1}{1-x-y} \to 3x+3y=2$ $\quad \frac{x}{y} = 7 \to x = 7y$



$3(7y)+3y=2 \to y=\frac{1}{12}$ $\quad x = 7\left(\frac{1}{12}\right) \to x = \frac{7}{12}$

$1-\frac{7}{12}-\frac{1}{12}=\frac{1}{3}$

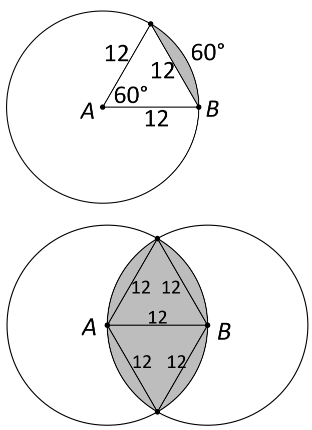
$a+3a=1 \to a=\frac{1}{4}$

Area of the shaded region = $\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)+\frac{1}{2}(1)\left(\frac{3}{4}\right)=\frac{5}{12}$

13. Let x represent the number of 50-cent increases. Revenue $=(60-10x)(4+0.5x)=$ $-5x^2-10x+240$. The maximum revenue will result when $x=-\dfrac{b}{2a}=-\dfrac{10}{2(5)}=-1$. Since x is negative, this will be a decrease of 50 cents. Maeby needs to charge \$3.50 to maximize revenue. The difference between \$5 and \$3.50 is \$1.50.

14. Use the arc length formula $\displaystyle\int_a^b \sqrt{r^2+(r')^2}\,d\theta$. Given $r = 1+\cos\theta$ we can find $r' = -\sin\theta$. Substituting these into the formula and simplifying gives $\int_0^{\pi} \sqrt{2+2\cos\theta}\,d\theta$. Using the identity $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$ results in the integral $\displaystyle\int_0^{\pi} 4\cos\frac{\theta}{2}\,d\theta$ which gives an answer of 4.

15.



A segment of the circle $=\dfrac{60°\cdot\pi\cdot 12^2}{360°}-\dfrac{12^2\sqrt{3}}{4}$

$=24\pi-36\sqrt{3}$

There are 4 of these segments $\rightarrow 96\pi-144\sqrt{3}$

Add the areas of the two equilateral triangles

$2\left(\dfrac{12^2\sqrt{3}}{4}\right)=72\sqrt{3}$

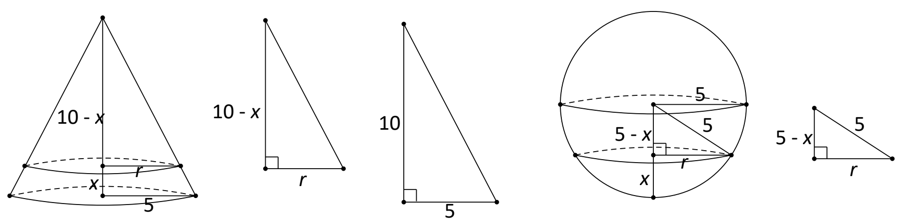
Area of the shaded region =

$96\pi-144\sqrt{3}+72\sqrt{3}$

$96\pi-72\sqrt{3}\rightarrow 24\left(4\pi-3\sqrt{3}\right)$

16. The common ratio is $-\dfrac{i}{3}$, so the sum is $\dfrac{9}{1-\left(-\dfrac{i}{3}\right)}\rightarrow\dfrac{9}{\dfrac{3+i}{3}}\rightarrow\dfrac{27}{3+i}\cdot\dfrac{3-i}{3-i}=\dfrac{81}{10}-\dfrac{27}{10}i.$

17.



$\dfrac{10-x}{10}=\dfrac{r}{5}\rightarrow x=10-2r$ $\qquad (5-x)^2+r^2=5^2\rightarrow x=10-2r$

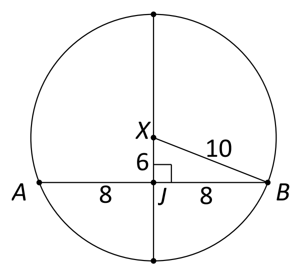
$25-10(10-2r)+(10-2r)^2+r^2=25\rightarrow r=4,\ x=2$

18. Separating the differential equation gives $\frac{dy}{0.1y+10} = dx$. Antideriving both sides and solving for y gives $y = ce^{\frac{x}{10}} - 100$. Using the initial condition gives a c value of 400 then evaluating ln 1024 or $\ln 2^{10}$ gives a result of 700.

19. The volume can be found by the scalar triple product, or just by find the determinant of the matrix formed by the components of each vector. $\begin{vmatrix} 3 & -5 & 1 \\ 0 & 2 & -2 \\ 3 & 1 & 1 \end{vmatrix} = 36.$

20. The two functions intersect at $y = \sqrt{2}$ and $y = -\sqrt{2}$. Using the washer method you can find the larger radius to be $4 - y^2$ and the smaller radius to be y^2. The integral can be set up as $2\pi \int_0^{\sqrt{2}} (4 - y^2)^2 - y^4 dy$ which solves to $\frac{64\pi\sqrt{2}}{3}$.

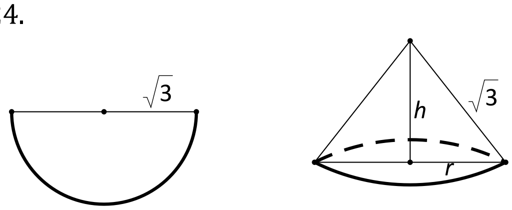
21. 16 inches



22. $y = \frac{3x+7}{x+2} = \frac{1}{x+2} + 3$, which is a left shift of 2 and a vertical shift up 3. For $y = x^2$ we have $y = (x+2)^2 + 3 = x^2 + 4x + 7$.

23. The MacLaurin representation for $f(x) = e^x$ is $\sum \frac{x^n}{n!}$. The representation for $f(x) = e^{-2x^2}$ would be $\sum \frac{(-2x^2)^n}{n!}$ so the degree 8 term would occur when n=4. $\frac{(-2)^4}{4!} = \frac{2}{3}$.

24.



$$\sqrt{3}\pi = 2\pi r \rightarrow r = \frac{\sqrt{3}}{2}$$

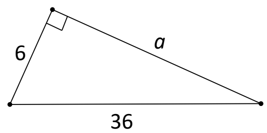
$$\left(\frac{\sqrt{3}}{2}\right)^2 + h^2 = \left(\sqrt{3}\right)^2 \rightarrow h = \frac{3}{2}$$

$$V = \frac{1}{3}\pi\left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{3}{2}\right) \rightarrow \frac{3}{8}\pi$$

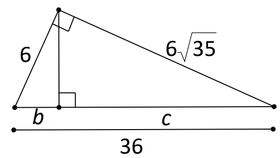
25. Since we have a semicircle, angle C is a right angle and the triangle is a right triangle. Let $AB = 2r$ and $\theta = \angle ABC$. This gives $BC = 2r\cos\theta$. The triangle area can then be found by $A = \frac{1}{2}(2r)(2r\cos\theta)(\sin\theta) = r^2(2\sin\theta\cos\theta) = r^2\sin 2\theta$. The area of the semicircle is $A = \frac{1}{2}\pi r^2$.

Equating these we get $2r^2\sin 2\theta = \frac{1}{2}\pi r^2 \rightarrow \sin 2\theta = \frac{1}{4} \rightarrow 2\theta = \sin^{-1}\frac{\pi}{4} \rightarrow \theta = \frac{1}{2}\sin^{-1}\frac{\pi}{4}$.

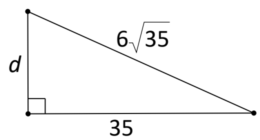
26.



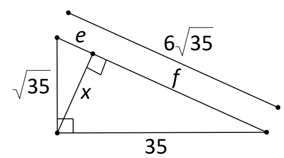
$$6^2 + a^2 = 36^2$$
$$a = 6\sqrt{35}$$



$$\frac{36}{6} = \frac{6}{b}$$
$$b = 1 \rightarrow c = 35$$



$$d^2 + 35^2 = \left(6\sqrt{35}\right)^2$$
$$d = \sqrt{35}$$



$$\frac{6\sqrt{35}}{\sqrt{35}} = \frac{\sqrt{35}}{e}$$
$$e = \frac{\sqrt{35}}{6} \rightarrow f = \frac{35\sqrt{35}}{6}$$

$$\frac{\frac{\sqrt{35}}{6}}{x} = \frac{x}{\frac{35\sqrt{35}}{6}} \rightarrow x = \frac{35}{6}$$

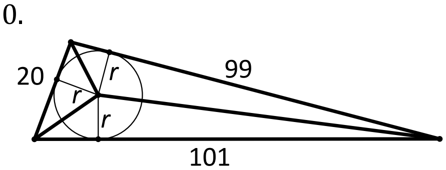
27. The integral needs to be split up at the point where the quadratic crosses under the x-axis. The integral can be split up into $\int_{-3}^{-1} -x^2 - 4x - 3\,dx + \int_{-1}^{2} x^2 + 4x + 3\,dx = \frac{4}{3} + \frac{54}{3} = \frac{58}{3}$.s

28. $f(x) = \left(1 + \cos\left(\sin^{-1}x\right)\right)\left(1 - \cos\left(\sin^{-1}x\right)\right) = 1 - \cos^2\left(\sin^{-1}x\right)$. Using Pythagorean identities, we

Get $f(x) = \sin^2\left(\sin^{-1}x\right) = \left(\sin\left(\sin^{-1}x\right)\right)^2 = x^2$. $f\left(-\frac{\sqrt{3}}{5}\right) = \left(-\frac{\sqrt{3}}{5}\right)^2 = \frac{3}{25} = 0.12$.

29. The slope of the tangent line is $y' = \frac{2\cos(x)}{2\sqrt{2\sin(x)+9}}$, $y'(0) = \frac{1}{3}$ and $y(0) = 3$. The equation of the tangent line would be $y = 3 + \frac{1}{3}(x - 0)$ and $y(-0.09) = 2.97$.

30.



$$A_\Delta = \sqrt{110(110-20)(110-99)(110-101)} = 990$$
$$\frac{1}{2}(101)r + \frac{1}{2}(99)r + \frac{1}{2}(20)r = 990$$
$$r = 9$$